Mathematics I (H)
Part I.

Theorem of Countable and Uncountable sets

Theorem - Prove that the Cartesian product of two Countable sets is Countable.

Proof: Let A and B be any two denumerable sets (or Countable sets).

Then we have to prove that $A \times B$ is also denumerable (or Countable)

Let $A = \{a_1, a_2, a_3, a_4, \ldots\}$ and
$B = \{b_1, b_2, b_3, b_4, \ldots\}$

then $A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3) \ldots$
$(a_2, b_1), (a_2, b_2), (a_2, b_3) \ldots$
$(a_3, b_1), (a_3, b_2), (a_3, b_3) \ldots \}$

The elements of $A \times B$ can be arranged is follows

$(a_1, b_1), (a_1, b_2)\ (a_1, b_3) \ldots (a_1, b_n) \ldots$
$(a_2, b_1), (a_2, b_2)\ (a_2, b_3) \ldots (a_2, b_n) \ldots$
$\ldots$
$(a_m, b_1), (a_m, b_2), (a_m, b_3) \ldots (a_m, b_n)$

For each fixed index i, We can Write

$A_i = \{(a_i, b_1), (a_i, b_2), (a_i, b_3) \ldots\}$.
$= \{x_{i1}, x_{i2}, x_{i3}, \ldots\}$.

Where x_{ip} Stand for (a_i, b_p)

$\therefore A \times B = \bigcup_{i=1}^{\infty} A_i$.

We know that Union of Countable family of Countable set is Countable.

$\therefore = \bigcup_{i=1}^{\infty} A_i$ is Countable as well as denumerable (or enumerable)

$\therefore A \times B$ is Countable.

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Theorem :– Show that the set of all irrational numbers is uncountable (or not enumerable).

Proof :– We suppose on the contrary the set of irrational numbers is enumerable.
Since the set of rational numbers is denumerable. Hence union of irrational numbers and rational numbers is enumerable because the union of two enumerable set is countable. But we know that union of set of irrational number and the set of rational numbers is the set of real numbers. Thus from this dicussion we have set of real numbers is denumerable. We know that the set of real numbers is not enumerable. This is the contradiction of our supposition.

Hence the set of rational is not denumerable.

Proved